Exercise 17

Show that the points $(0, -2, -1)$, $(1, 4, 0)$, $(2, 10, 1)$ do not determine a unique plane.

Solution

Notice that these three points are collinear. An equation of the line that has all three is

$$\mathbf{y}(t) = (t, 6t - 2, t - 1) = (t, 6t, t) + (0, -2, 1) = (1, 6, 1)t + (0, -2, 1).$$

Two planes that contain this line, for example, are

$$4x + y - 10z = 8$$

and

$$x - y + 5z = -3,$$

which can be verified by plugging in $x = t$, $y = 6t - 2$, and $z = t - 1$.